# PER UNIT SYSTEM

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# INTRODUCTION

- In power system different equipment with different voltage and power levels are connected together through various step up and step down transformers.
- The presence of various voltage and power level causes problem in finding out the current or voltage at different points in the network.
- To solve this problem, all the system quantities are converted into a uniform normalized platform which is called per unit system.
- In a per unit system each system variable or quantities is normalized with respect to its own base value.

$$Quantity_{pu} = \frac{Quantity_{Actual}}{Quantity_{Base}}$$

- Both the actual value and base value is expressed in the same units so the per unit value is dimension less.

# INTRODUCTION

- In power system, out of four system quantities, KVA, KV, current and impedance, only two are commonly known and are convenient to be select as the base values, i.e. KVA & KV.
- The base of current & impedance may be calculated as

$$Current_{base} = \frac{Power_{base}}{Voltage_{base}} = \frac{Base\ KVA}{Base\ KV}$$

$$Impedance_{base} = \frac{Voltage_{base}}{Current_{base}} = \frac{(Voltage_{base})^2}{KVA_{base}}$$

- For the circuit containing transformers, it is convenient to select the same KVA base for both the sides.

# NUMERICAL PROBLEM

- **Problem statement –** A 230 KV transmission line has a series impedance of 4 + j 60 ohm and a shunt admittance of  j 2 * 10-3 S. Using 100 MVA and the line voltage as base value, calculate per unit impedance. (Answer – (7.56 + j 113.4) * 10-4 pu)
- **Problem statement –** A 230 KV transmission line has a series impedance of 4 + j 60 ohm and a shunt admittance of  j 2 * 10-3 S. Using 100 MVA and the line voltage as base value, calculate per unit admittance. (Answer – j 1.058 pu)
- **Problem Statement –** If a 250MVA,11/400 KV, Three –Phase power Transformer has leakage reactance of 0.05 pu on the base of 250 MVA and the primary voltage of 11KV,then determine the actual leakage reactance of the Transformer refereed to the secondary side of 400KV. (Answer – 32 ohms)

# BASE TRANSFORMATION

- It is usually necessary to transform the per unit impedance from one set of base to new set of base value.
- This is done as follows.

$$Z_{pu,new} = Z_{pu,old} \times \left(\frac{base\ KV_{old}}{base\ KV_{new}}\right)^2 \times \frac{base\ KVA_{new}}{base\ KVA_{old}}$$

# NUMERICAL PROBLEMS

- **Problem statement –** The per unit impedance of a transmission line is X p.u. Determine the new per unit impedance is the base voltage is tripled and base MVA is doubled. (Answer – 2/9 times X)
- **Problem Statement –** A synchronous generator rated 11 KV, 50 MVA has a per unit impedance of 0.2 pu on its own base. Determine the impedance when calculated at 22 KV, 150 MVA base. (Answer – 0.15 pu)
- **Problem Statement –** The per unit impedance of a synchronous machine is 0.242 pu. If the base voltage is increased to 1.1 times, determine the new per unit impedance. (Answer – 0.200 pu)
- **Problem Statement –** The Direct axis reactance of a synchronous generator is given as 0.4 pu based on the generators name plate rating of 10 KV, 75MVA. If the base for calculation is 11KV,100MVA, determine the pu value of Generator on the new base. (Answer – 0.44)

# NUMERICAL PROBLEM

- **Problem statement –** A portion of a power system consists of two generator in parallel, connected to a step up transformer that links them with a 230 KV transmission line. The rating of these components are
  - Generator (G1) – 100 MVA; 12% reactance (Answer – 1.8%)
  - Generator (G2) – 5 MVA; 8% reactance (Answer – 24%)
  - Transformer – 15 MVA; 6% reactance (Answer – 6%)
  - Transmission line – 4 + j60; 230 KV (Answer – 0.113 + j 1.7)

where the percent reactance are computed on the basis of individual component rating. Express the reactance and impedance with 15 MVA as the base value

# PER UNIT IMPEDANCE OF A TRANSFORMER

- The per unit equivalent impedance of a 2 – winding transformer is the same weather the calculation is made from the high voltage side or the low voltage side.
- Consider a single phase transformer with total series impedance $(Z_{e1})$; voltage $(V_1)$ and current $(I_1)$.
- If the applied voltage and current are assumed to be the base values; then the per unit impedance in given as

$$(Z_{e1})_{pu} = \frac{Z_{e1}}{Z_b} = \frac{Z_{e1} \times I_1}{V_1}$$

- Similarly, if the total impedance referred to secondary side is $(Z_{e1})$; and the base voltage and current for the secondary side is $(V_2)$ and $(I_2)$ respectively then the base impedance is given as

$$(Z_{e2})_{pu} = \frac{Z_{e2}}{Z_b} = \frac{Z_{e2} \times I_2}{V_2}$$

# PER UNIT IMPEDANCE OF A TRANSFORMER

- We know that, the total impedance referred to secondary side is

$$Z_{e2} = Z_{e1} \times \left(\frac{N_2}{N_1}\right)^2$$

$$V_2 = \frac{N_2 \times V_1}{N_1}$$

$$I_2 = \frac{N_1 \times I_1}{N_2}$$

- Therefore,

$$(Z_{e2})_{pu} = Z_{e1} \times \left(\frac{N_2}{N_1}\right)^2 \times \frac{N_1 \times I_1}{N_2} \times \frac{N_1}{N_2 \times V_1} = \frac{Z_{e1} \times I_1}{V_1}$$

$$(Z_{e2})_{pu} = (Z_{e1})_{pu}$$

# NUMERICAL PROBLEM

- **Problem statement** – A 5 KVA 400/200 Volts, 50 Hz single phase transformer has primary & secondary leakage reactance each of 2.5 ohms. Determine
  - Total impedance referred to primary side (Answer – 12.5 ohms)
  - Per unit impedance referred to primary side (Answer – 0.390625 pu)
  - Total impedance referred to secondary side (Answer – 3.125 ohms)
  - Per unit impedance referred to secondary side (Answer – 0.390625 pu)
  - Per unit impedance when commutated at individual sides (Answer – 0.390625 pu)

# PER UNIT QUANTITIES IN THREE PHASE SYSTEM

- The per unit impedance referred to either side of a three phase transformer is the same irrespective of the tree phase connection whether they are delta/delta; star/star; or delta/star.
- In a star connection

$$V_l = \sqrt{3}.V_p;\ V_{lb} = \sqrt{3}.V_{pb}$$

$$I_l = I_p;\ I_{lb} = I_{pb}$$

$$(V_l)_{pu} = \frac{V_l}{V_{lb}} = \frac{\sqrt{3}.V_p}{\sqrt{3}.V_{pb}} = \left(V_p\right)_{pu}$$

$$(I_l)_{pu} = \frac{I_l}{I_{lb}} = \frac{I_p}{I_{pb}} = \left(I_p\right)_{pu}$$

# PER UNIT QUANTITIES IN THREE PHASE SYSTEM

- In delta connection

$$I_l = \sqrt{3}.I_p;\ I_{lb} = \sqrt{3}.I_{pb}$$

$$V_l = V_p;\ V_{lb} = V_{pb}$$

$$(I_l)_{pu} = \frac{I_l}{I_{lb}} = \frac{\sqrt{3}.I_p}{\sqrt{3}.I_{pb}} = (I_p)_{pu}$$

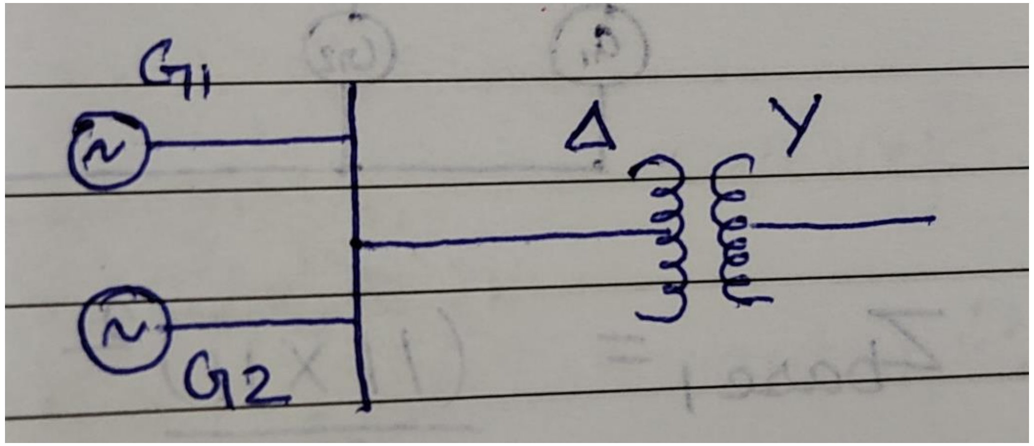
$$(V)_{pu} = \frac{V_l}{V_{lb}} = \frac{V_p}{V_{pb}} = (V_p)_{pu}$$

# SINGLE LINE DIAGRAM

- A single line diagram or one line diagram is a graphical representation of the essential of a system in a most simplified form.
- In the impedance diagram, the different components of the power system are replaced by their equivalent circuit. The synchronous generator is replaced by a constant voltage source by equivalent circuit. The transmission line is replace by nominal pi equivalent circuit.
- In many studies the resistance of generator, resistance of transformer winding, resistance of transmission line, line charging and magnetizing circuit of transformer are neglected. Under this condition the impedance diagram is represented by a reactance diagram.

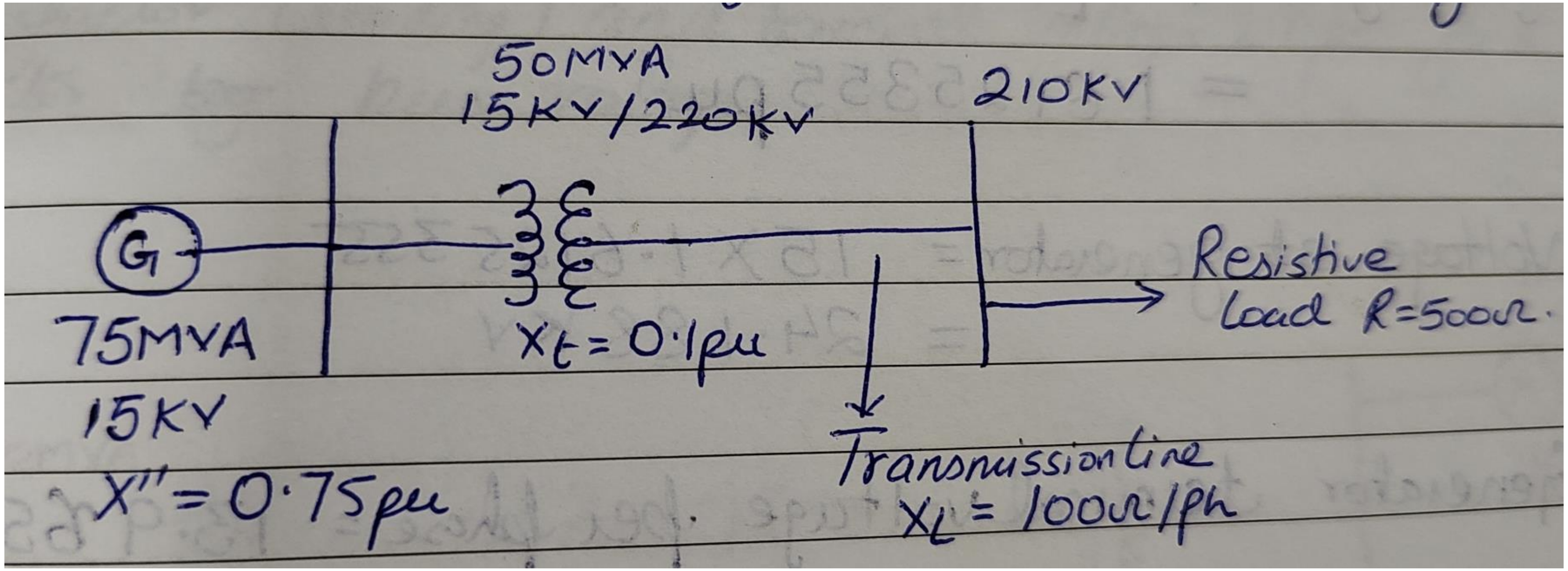
# NUMERICAL PROBLEM

- **Problem statement –** Assume the base of 25 MVA and 60 MVA; calculate the impedance in ohms between the generator and the output terminals of the transformer for the system shown in the figure. The specification of generator and transformer are –
  - Generator 1 – 30 MVA, 11 KV, X=0.20 pu
  - Generator 2 – 25 MVA, 11 KV, X=0.25 pu
  - Transformer – 60 MVA, 11 KV (delta)/66 KV (star), X=0.10 pu (Answer – j 0.685678)



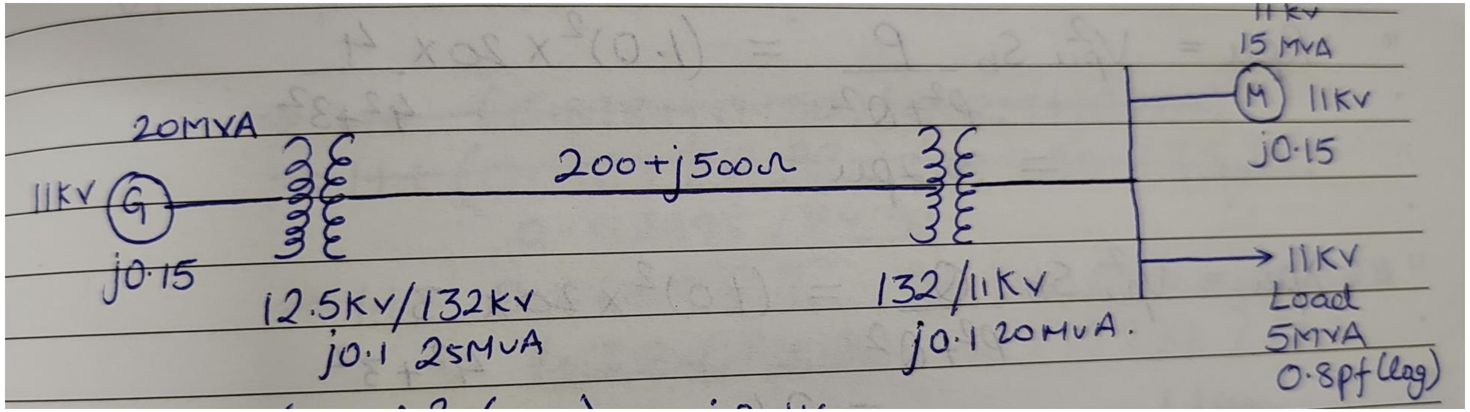
# NUMERICAL PROBLEM

- **Problem statement** – For the system shown in the Figure; determine the generator voltage. (Answer – 24.188 KV)



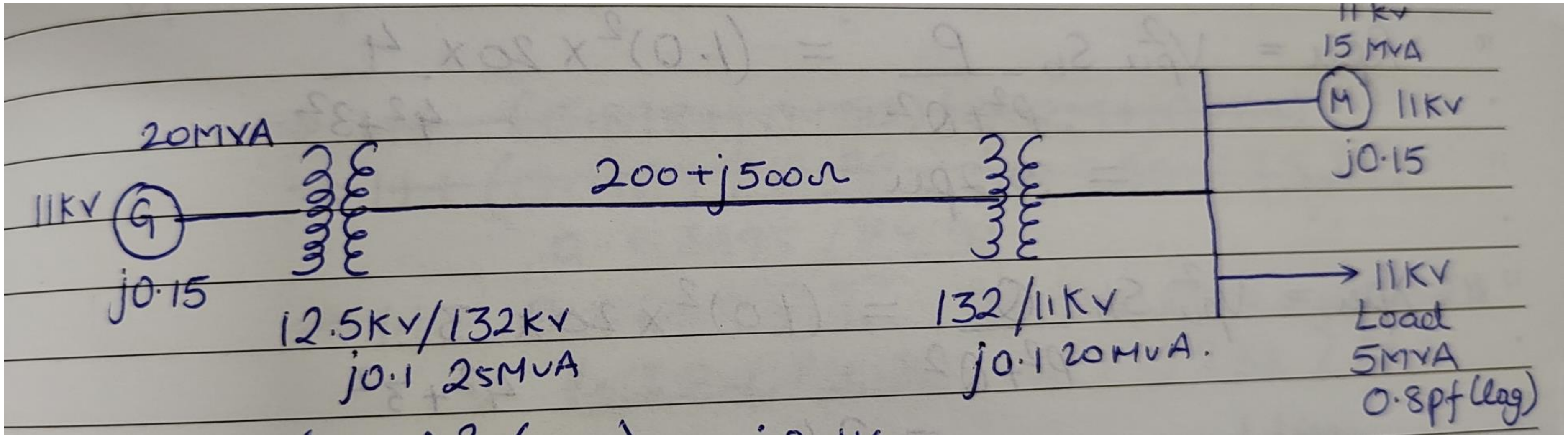
# NUMERICAL PROBLEM

- **Problem Statement –** Figure shows a two machine system. Draw the impedance diagram for the system. Choose a base voltage of 132 KV for the transmission line and base power of 20 MVA. The rating are as follows.
  - Generator – 20 MVA, 11 KV, X = 0.15 pu
  - Motor – 15 MVA, 11 KV, X= 0.15 pu
  - Transformer 1 – 25 MVA, 12.5 KV (delta)/132 KV (star)
  - Transformer 2 – 20 MVA, 132 KV (star)/ 11 KV (delta)
  - Line – 200 + j 500 ohms
  - Static load – 5 MVA, 0.8 lagging p.f.



# NUMERICAL PROBLEM

- **Problem statement** – In the previous question, if the motor is a synchronous machine drawing 15 MVA at 0.9 power factor (leading) and terminal voltage 1.1 pu. Find the generator bus voltage. (Answer - 1.3238; 28.71 degree)



# ADVANTAGE OF PER UNIT SYSTEM

- Manufactures usually specify the impedance value of equipment in per unit of the equipment rating. If any data is not available, it is easier to assume its per unit valve than its numerical values.
- When expressed in per unit, system parameters tends to fall in relatively narrow numerical range.
- Per unit data representation yield important information about relative magnitude.
- Power system contains a large number of transformer. The ohmic value of impedance as referred to primary. However if he base value are selected properly the per unit value is same for both the sides.
- The transformer connected in 3 phase circuit do not affect the per unit value of impedance although the base voltage on both sides do depend on the connections.